ISI – Bangalore Center – B Math - Physics I – Mid Term Exam
Date: 5th March 2015. Duration of Exam: 3 hours
Total marks: 35

ANSWER ALL QUESTIONS

Q 1 [Total Marks: 2+1+1+2+2=8]

For this question ONLY THE ANSWERS NEED TO BE PROVIDED. Calculations need NOT be shown. Please just write the final answer, as appropriate.

Q 1a. A particle of mass *m* starts from starts from rest at $x_0 > 0$ in a force field $\vec{F} = -(k/x^3)\hat{x}, k > 0$. The time taken to reach the origin is proportional to
i) $x_0^{1/2}$, ii) x_0 , iii) x_0^2 , iv) x_0^3 (Choose one answer)

Q 1b. In the same problem as above, state if the following statement is True or False,

As the particle approaches the origin, its speed increases without bound.

Q 1c. A particle is moving in such a way that its acceleration vector is at an acute angle with the velocity vector. Which is the following statement is true?
i) kinetic energy increasing , ii) kinetic energy decreasing ii) kinetic energy remains constant

Q1d. A particle moves in a potential $V(x) = k\,|x|^n$. where n is an even integer greater than 1. Can the particle if given sufficient energy escape to infinity? Answer Yes or No. Will your answer change if n is an odd integer greater than 1? Answer Yes or No.

Q1e. Which one among the three Kepler'r Laws of planetary orbits will remain valid if the inverse square force is replaced by an arbitrary central force? State the law.

Q2. [Total Marks: 9]

The force on a particle of mass *m* moving in one dimension is given by $\vec{F}(x) = 2cx(x^2 - a^2)\hat{x}$ where *c* and *a* are both positive constants. Assume that the total energy of the particle at rest at $x = 0$ is $-\frac{1}{2}ca^4$.

Q2a. Sketch the potential energy of the particle. What are the values of x for stable equilibrium and what are the values of x for unstable equilibrium?

Q2b. If E is such that $-\frac{1}{2}ca^4 < E < 0$ is the motion of the particle bounded or unbounded?

Q2c. Let at *t=0*, the particle be at *x = a/2* with zero total energy, moving towards the origin. Calculate the time taken to reach *x = - a/2*.

Q2d. How long will the particle take to reach *x = -a* ? (this can be answered without any long calculation)

Q3. [Total Marks: 9]

Q3a.

The equation of motion of a damped harmonic oscillator is $\frac{d^2x}{dt^2} + 2K\frac{dx}{dt} + \Omega^2 x = 0$ where *K >0*. Consider the case $K > \Omega$. Prove that regardless of the initial position $x_0 \neq 0$ of the particle, if the initial speed is zero then the particle will never cross the origin. (You can use the solutions to the above equation without deriving it)

Q3b.

A particle of mass *m* moves along the *x*-axis and is acted upon by the restoring force $-m(n^2 + k^2)x$ and the resistance force $-2mk\dot{x}$ where *n, k* are positive constants. If the particle is released from rest at *x=a*, show that in subsequent motions

$$x = \frac{a}{n}e^{-kt}(n\cos nt + k\sin nt).$$

Find how much the particle travels before it next comes to a stop.

Q4. [Total Marks: 9]

A particle of mass *m* moves under the attractive inverse square field $\vec{F} = -(m\gamma / r^2)\hat{r}$.

Q4a. Write the energy equation for the system using polar coordinates.

Q4b. Starting from the energy equation, show that the equation satisfied by the apsidal distances is $2Er^2 + 2\gamma r - L^2 = 0$ where *E* and *L* are the specific total energy and angular momentum of the particle.

Q4c. Assume now that E <0, and the orbit is an ellipse with the origin as the focus. By considering the sum and products of the roots of the above equation, show that for an elliptic orbit $E = -\frac{\gamma}{2a}$, and $L^2 = \gamma\frac{b^2}{a}$ (You can use the polar equation of an ellipse $\frac{1}{r} = \frac{a}{b^2}(1 + e\cos\theta), e = 1 - \frac{b^2}{a^2}, a > b$)